Applied Mathematics and Statistics

Common Qualifying Examination Part B

in Computational Applied Mathematics

Spring 2017 (January)

(Closed Book Exam)

Please solve 3 out of 4 problems for full credit.

Indicate below which problems you have attempted by circling the appropriate numbers:

Part B: 1 2 3 4

NAME ______________________________

Start each answer on its corresponding question page. Print your name, and the appropriate question number at the top of any extra pages used to answer any question. Hand in all answer pages.

Date of Exam: January 23, 2017

1) Consider the following 3rd order linear inhomogeneous differential equation

$$y''' - 3y'' + 2y' = f(x).$$

Solve the corresponding Green's function problem and express a particular solution to this equation in terms of the Green's function.

2) (a) Find a second order linear homogeneous differential equation that has only one regular singular point on the entire real line, including infinity, and no other singular point.

(b) What is the general form of an nth-order linear homogeneous differential equation that is known to have just two regular singular points on the entire real line, including infinity, and no other singular points? Prove your statement.

3) Consider the Householder transformation H associated with a vector $a \in \mathbb{R}^n$, where $H = I - 2ww^T$ for some unit vector w such that $Ha = \beta e_1$.

   (a) (3 points) If H is real, show that $\beta = \pm\|a\|_2$, and explain how to choose the sign.

   (b) (4 points) Given a and β, explain how to construct w.

   (c) (3 points) Given a vector $v \in \mathbb{R}^n$, how many flops are required to compute Hv with an optimal algorithm?

4) If $P \in \mathbb{R}^{n \times n}$ is a permutation matrix.

(a) (4 points) Show that P is unitarily diagonalizable.

(b) (6 points) Show that all the eigenvalues of P are unit roots of 1.